Foundations of Mechanism Design: Vickery-Clarke-Grove Mechanism

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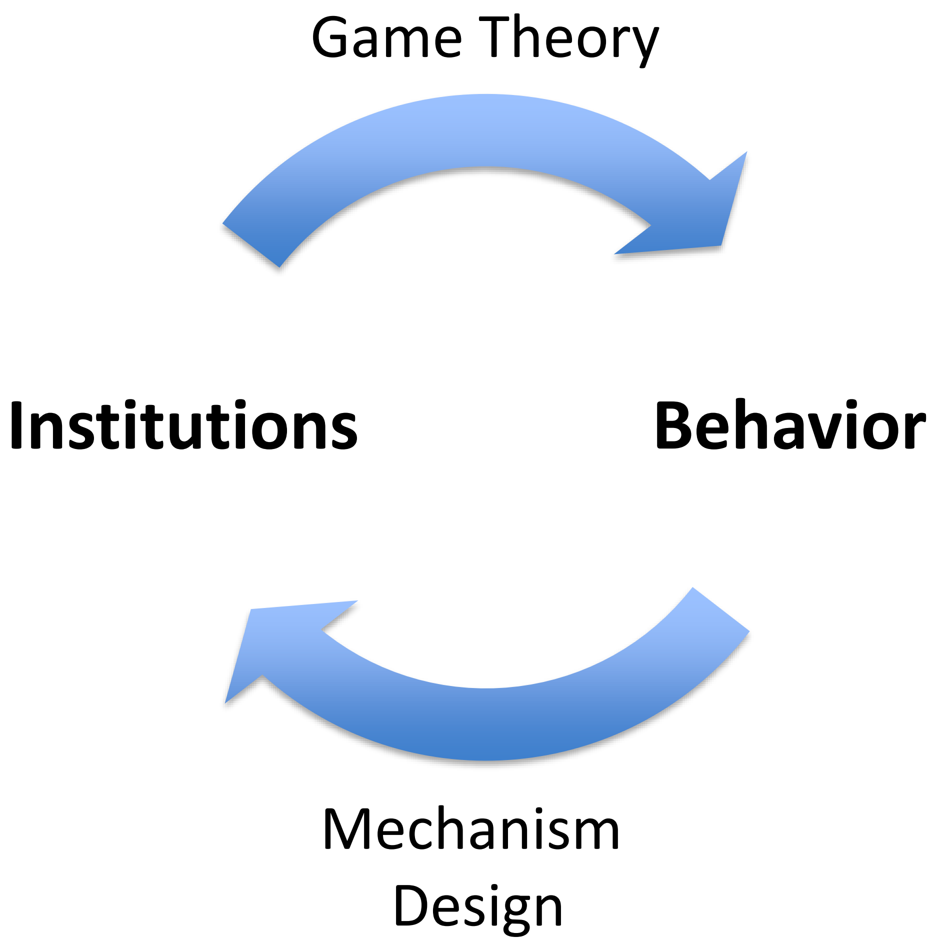
Mechanism Design



The Big Problem

- How to implement “efficient” allocation in an environment where participants have private information about their preferences?
  - Auctions: Spectrum, ads, oil, construction, …
  - Decision to build a public project
  - Resolution of dispute between several parties

Incentives

- Participants can misrepresent their preferences:
  - A bank can overstate its need for bailout
  - A buyer can understate its value, hoping to get lower price
  - A network provider overstate its costs, hoping to get higher price

A General Setting

- **N** players and a market designer (player 0)
- Each player has a type $(t_1, \ldots, t_N)$
  - Private information
- **Outcome** is (x, **p**), where $x \in X$ is a decision and **p** = $(p_1, \ldots, p_n)$ are participants payments.
- $u_i = v_i(x) - p_i$

Limitations

- Participants know their own values
- Preferences are over final outcome, not the process
- There are no limits to making transfer payments.
- Preferences are quasi-linear in money.

Two Key Criteria

- A decision is **efficient** if

$$x \in \operatorname{argmax}_{x \in X} \sum_{j \in N} v_j(x)$$

- A mechanism is **strategy-proof** if truthful reporting of preferences is players' optimal strategy (in expectation), regardless of other players' reports.

- Can we design an **efficient & strategy-proof** mechanism?

Why SP is desirable?

- Simplicity, detail-free (Wilson, 1975)
- No gain from spying
- No need to be sophisticated (Pathak & Sonmez, 2006)
- No advantage due to outside options (van Dijk & Akbarpour, 2016)

Vickery-Clarke-Grove (VCG)

- The market designer announces:

Report your utilities for each decision, I will pick decision x^* *that* ***maximizes total values assuming you're honest****, and charge each player according to the following rule:*

$$p_i = \max_{x \in X} \sum_{j \in N/i} v_j(x) - \sum_{j \in N/i} v_j(x^*)$$

Vickery-Clarke-Grove (VCG)

- What's going on?

Other players total utility, given i's report

$$p_i = \max_{x \in X} \sum_{j \in N/i} v_j(x) - \sum_{j \in N/i} v_j(x^*)$$

Other players total utility, in an imaginary world that i didn't exist

- i's payment: Her **externality** on others!

Example 1: Single-item Auction

- 1 item to sell, N buyers.
- Buyers have some private value v_i
- VCG auction:
  - *Report utilities, highest bidder wins, pays the second highest bid. (why it's VCG?)*

Example 2: Bilateral Trade

- 1 buyer (value v_b) and one seller (value v_s)

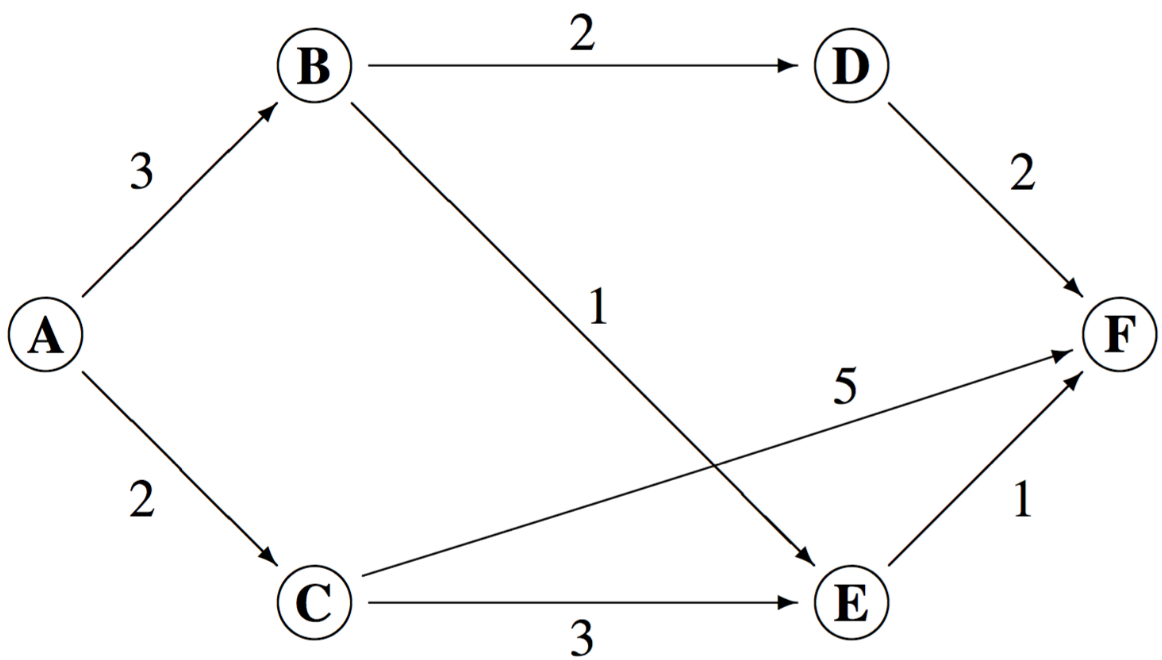
- No trade if $v_b < v_s$ & payments = 0

- Trade if $v_b > v_s$

- $P_b = v_s – 0 = v_s$
- $P_s = 0 – v_b = - v_b$

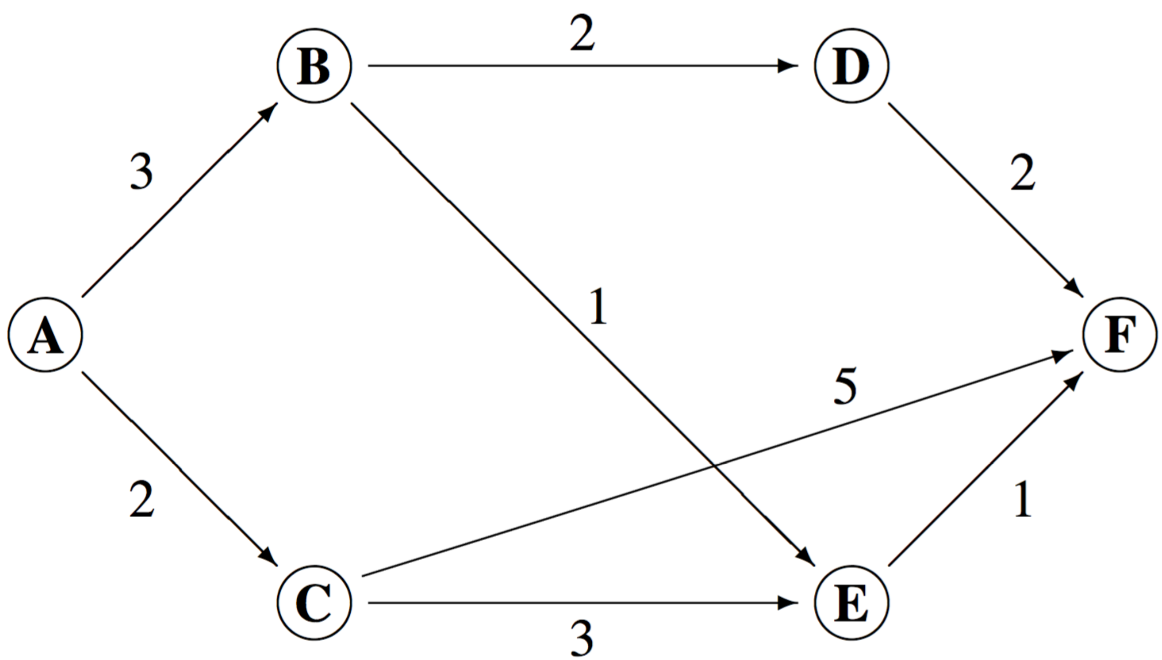
- Not budget balanced! (planner should subsidize!)

Example 3: Selfish routing



Efficient path: ABEF

Example 3: Selfish routing



AB payment = (-6) – (-2) = -4

BE payment = (-6) – (-4) = -2

EF payment = (-7) – (-4) = -3

VCG is Strategy-Proof

- **Theorem**: VCG is SP and efficient.
- **Proof.** Suppose players report their valuations as r_i (x). The planner aims at maximizing

$$r_i(x^*) + \sum_{j \in N/i} r_j(x^*)$$

While you like to maximize

$$v_i(x) - \qquad\qquad - \sum_{j \in N/i} r_j(x))$$

Why VCG is lovely?

- Can be used in any environment where payments are allowed.
- Can be used in “package” bidding
- Outcome is efficient
- It’s strategy-proof

So, why it’s so lonely in practice?!

Lovely but Lonely VCG: Privacy

- Bidders can have privacy concerns and prefer not to reveal, at least when they loose.
- VCG is the *worst* mechanism in this regard

Lovely but Lonely VCG: Low Revenue

- Two items: A & B
- Package bidder: values both at 10
- Two individual bidders: Each values each item at 9 (and values package at 9).
- Efficient: award item to individual bidders
- Payments: 1 for each bidder
- Revenue: 2 (could be 10 by giving item to package bidder!)

Lovely but Lonely VCG: Collusion

- Two items: A & B
- Package bidder: values both at 10
- Two individual bidders: Each values each item at 2 (and values package at 2).
- Honest bidding: package bidder wins
- Individual bidders can collude and jointly report value 9 for each item.
- They win, and pay 1 !

Lovely but Lonely VCG: “Shill” Bids

- Two items: A & B
- Package bidder: values both at 10
- **One** individual bidder: Values each item (&package) at 9.
- Honest bidding: package bidder wins and pays 9
- Individual bidder can enter the auction as “two” bidders, bid 9 for each item, win, and pay 1 for each item!

Lovely but Lonely VCG: Budgets

- Two items: A & B
- Bidder values A at 200 and B at 100, budget 150.
- Can't bid true values and be sure that budget constraint is met.
- It's generally complex to bid with budget constraint in a VCG mechanism

Lovely but Lonely VCG: Computations

- To calculate each bidder payment, we should solve two optimization problems!

- In large markets (like FCC incentive auction), optimization problems are NP-hard.

- We may be able to solve them in several weeks (or months!), but what if we need to announce payments very quickly?

Lovely but Lonely VCG: Cheating

- The planner can always “cheat”
- Example: in a second-price auction, the planner can cheat and report a higher bid as the second highest bid and charge the winner more!
- The planner **cannot** do this in a 1^{st} price auction. (Why?)
- What kind of mechanisms are “credible”?

“Credible” Mechanisms

- **Akbarpour & Li, 2016**: “Credible mechanism design” (work in progress)

- A market designer is the “center of communications” with “bilateral commitment”.

- The market designer can cheat if not measurable.

Credible & Optimal Mechanisms

- **Myerson (1981)** Any mechanism that sets the right reserve price and sells the item to the highest value bidder is *optimal* (maximizes revenue).
  - 1st price auction, 2nd price auction, 3rd price auction, all-pay auction, half-pay auction, … .

- **Theorem (Akbarpour & Li, 2016)**: In the class of sealed-bid (static) auctions, *the only* optimal and credible auction is the 1st price auction.

Market Design: Future

- *Behavioral* game theory showed that classic game theory isn’t quite predictive.

- *Behavioral Market Design* is going to be an important future direction.
  - Li, 2016 . “Obviously Strategy-proof Mechanisms.”
  - …

Market Design: Future

- *Exact* computation of optimization problems is impossible in many real-world settings.

- *Algorithmic Market Design* is going to be an important future direction.
  - Milgrom & Segal (2015) “Deferred Acceptance Auctions”
  - Akbarpour, Li, Oveis Gharan (2015), “Thickness and Information in Dynamic Matching Markets.”
  - …

Thank You ☺

Knapsack Problem

- A container (“knapsack) with size S
- N items, each with size s_i and value v_i
- Goal: pack items into knapsack to maximize total value.

→ NP-Complete

Dantzig's Greedy Algorithm

- Sort items by v_i/s_i
- Put items according to value per size in knapsack (if there's space remaining for that item).
- Stop when no more items remains

Greedy: It can be *very* Bad

- Let's construct an example together…

Modified Greedy

- Sort items by v_i/s_i

- Put items according to value per size in knapsack (if there’s space remaining for that item).

- Stop when no more items remains

- **Report: The outcome of Greedy OR the highest value item**

Approximation Bound

- ***Theorem***: The value of the modified Greedy performance is at least 50% of the solution of the (NP-Complete) optimum of the Knapsack problem.

Approximation Bound

- ***Theorem***: If each item's size is at most *f%* of the knapsack size, then the value of the Greedy algorithm is at least *(1-f)%* of the solution of the (NP-complete) Knapsack problem.

An Auction to Sell Space

- Private values, but public sizes.

- Goal: Design an (approximately) efficient mechanism, which is SP.

- VCG?
  - Lovely! But computationally not feasible!

The LOS Auction

- *Lehman, O'Callaghan, and Shoham (2002)* auction:

- Ask agents to report values
- Assuming they are honest, run the greedy algorithm
- Those who do not enter → no payment
- Those who enter the knapsack:

$$p_i = \inf\{v_i' \mid i \text{ is still included in the knapsack}\}$$

The LOS Auction

- **Theorem**: The LOS Greedy auction is SP.

- **Proof**.

  – Step 1: Greedy is *monotonic.*

  – Step 2: Any monotonic auction is SP.
  – Why? Intuition is same as 2nd price auction!

LOS: Pros and Cons

- Biggest drawback of Greedy LOS Auction:
  - It’s inefficient

- Nice features of Greedy LOS Auction:

  - It is *obviously* SP.

  - It can be computer in polynomial time

  - It is group-SP

  - Cheating isn’t a concern